

where the general expression for  $Q$  is

$$Q = 1.13 \left[ \int_{\theta_w}^1 \frac{\rho k}{\rho_e k_e} d\theta \int^1 \frac{c_p^*}{c_{pe}} d\theta^* \right]^{1/2} \quad \theta \equiv \frac{T - T_0}{T_e - T_0} \quad (4)$$

For Edwards and Tellep's power-law properties

$$k/k_e = \theta^r \quad \rho/\rho_e = \theta^s \quad c_p/c_{pe} = \theta^t \quad (5)$$

the expression for  $Q$  becomes

$$Q = \frac{1.13}{(t+1)^{1/2}} \left[ \frac{1 - \theta_w^{r+s+1}}{r+s+1} - \frac{1 - \theta_w^{r+s+t+2}}{r+s+t+2} \right]^{1/2} \quad (6)$$

$Q$  is related to the  $\Omega'$  of Ref. 1 by Eq. (3), and  $\Omega$  is related to  $\theta$  by

$$\Omega = \theta^r + s + 1 \quad (7)$$

Thus, for cold walls, Eq. (6) is a correlation formula for the curves of Fig. 1 of Ref. 1. The expression for  $Q$  has been found accurate to within 3% for  $0 < \theta_w < 1$ ,  $\frac{1}{2} < r < \frac{5}{2}$ ,  $s = -1$ ,  $t = 0$  and should be equally accurate for other values of  $s$  and  $t$ .

Reference 4 also contains expressions for upper and lower bounds on the exact value of  $Q$ , involving integrals similar to the ones given in Eq. (4), as well as an analysis for stagnation points at Prandtl numbers around unity.

#### References

<sup>1</sup> Edwards, D. K. and Tellep, D. M., "Heat transfer in low Prandtl number flows with variable thermal properties," *ARS J.* **31**, 652-654 (1961).

<sup>2</sup> Jepson, B-M., "Heat transfer in a completely ionized gas," Magnetogasdynamics Lab. Rept. 61-7, Dept. of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. (July 1961).

<sup>3</sup> Fay, J. A. and Kemp, N. H., "Theory of end wall heat transfer in a monatomic gas, including ionization effects," Research Rept. 166, Avco-Everett Research Lab., Everett, Mass. (1963).

<sup>4</sup> Kemp, N. H., "Approximate analytical solution of similarity boundary layer equations with variable fluid properties," Publ. 64-6, Fluid Mechanics Lab., Dept. of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. (1964).

## Reply by Authors to N. H. Kemp

D. K. EDWARDS\*

University of California, Los Angeles, Calif.

AND

D. M. TELLEP†

Lockheed Aircraft Corp.,

Sunnyvale, Calif.

IN the preceding note Kemp makes the point that the solution to the momentum equation for inviscid flow is  $u = u_e$  only when the fluid has constant density or when the flow is over a flat plate. The point is obvious and needs no laboring. Edwards and Tellep in their 1961 note<sup>1</sup> ruled out other cases with the statement "Only for the flat plate with  $du_e/dx = 0$  and  $dP/dx = 0$  will the analysis presented hold for fluids with pressure dependent properties." What was intended was to rule out the compressible fluid. The authors simply overlooked ruling out the hypothetical fluid which is incompressible but capable of significant thermal expansion or contraction. It is this hypothetical fluid that is the concern of Kemp, not to treat, but to eliminate from consideration.

An observation on the original note and the present one by Kemp is that the original exact results bear out the use of a

Received June 22, 1964.

\* Associate Professor, Department of Engineering. Member AIAA.

† Manager, Launch and Entry Thermodynamics, Missile and Space Division. Member AIAA.

reference temperature as suggested by Rubesin and Johnson<sup>2</sup> and Eckert.<sup>3</sup> Derivation of an expression for the reference temperature is facilitated by fitting the exact solution for  $\Omega'(0)/(1 - \Omega_w)$  with  $(2/\pi^{1/2})[(1 - am) + am\Omega_w]$ , where  $a$  is adjusted for the best fit.<sup>4</sup> When  $T_w$  is not greatly different from  $T_e$ , a linearized result gives very nearly the result recommended by Eckert. The reference temperature concept gives results as useful as the approximate relation of Kemp or the previous exact results.

#### References

<sup>1</sup> Edwards, D. K. and Tellep, D. M., "Heat transfer in low Prandtl number flows with variable thermal properties," *ARS J.* **31**, 652-654 (1961).

<sup>2</sup> Rubesin, M. W. and Johnson, H. A., "A critical review of skin friction and heat transfer solutions of the laminar boundary layer on a flat plate," *Trans. Am. Soc. Mech. Engrs.* **71**, 383 (1949).

<sup>3</sup> Eckert, E. R. G., "Engineering relations for heat transfer and friction in high velocity laminar and turbulent boundary layer flow over a surface with constant pressure and temperature," *Trans. Am. Soc. Mech. Engrs.* **78**, 1273 (1956).

<sup>4</sup> Zwick, E., unpublished solution to homework problem (1962).

## Comment on "Method for the Determination of Velocity Distribution in a Thin Liquid Film"

R. M. NEDDERMAN\*

University of Cambridge, Cambridge, England

IN a recent note, Persson<sup>1</sup> described a method of measuring velocity profiles in thin films of liquid. This was done by photographing, with a finite exposure, small particles moving with the fluid. The velocities were obtained by measuring the length of the resultant images. The frequency with which various velocities were observed was determined, and hence, by assuming that the particles were uniformly distributed throughout the liquid, the velocity profile could be calculated.

Recent work by Jeffrey<sup>2</sup> has shown that in full tube Poiseuille flow there is a marked tendency for solid particles to migrate across the streamlines and take up a position at about one-third of the radius from the axis. Although this complete migration is admittedly slow, the region near the tube wall rapidly becomes deficient in particles.

There is no reason to assume that this effect does not occur in film flow. If it does, it will produce a particle-deficient layer near the wall, and so velocity profiles determined by Persson's method would have an error in the distance scale. This would be in such a direction as to give a positive velocity at the wall. Inspection of Persson's curves shows that this tendency does in fact exist, and that the unexpected kink in his velocity profiles arises because the profile has been forced to pass through  $V = 0$  at  $\delta/\delta_0 = 0$ .

Some years ago the author<sup>3-5</sup> published an account of a method of measuring velocity profiles in thin films of liquid which avoids the difficulties inherent in Persson's method. Instead of photographing the particles with one camera only, two were used, inclined at an angle of about 40° to each other. The velocities were determined by the same method as Persson used, but the distances from the wall were found from the stereoscopic effect, i.e., the relative positions of the images on the two films. This technique is, in principle, the same as that used for the determination of altitude in aerial surveying. It was found possible by this technique to measure distances across the film to an accuracy of about  $\pm 0.001$  cm,

Received May 25, 1964.

\* Praelector and Fellow of Darwin College, University Demonstrator in Chemical Engineering.